

Forecast, Solution, and Rolling Horizons in Operations Management Problems: A Classified Bibliography

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We present a classified bibliography of the literature in the area of forecast, solution, and rolling horizons primarily in operations management problems. Each one of over 200 selected papers is categorized on five dimensions that identify the horizon type, the model type (deterministic or stochastic), the sources of the horizon, the methods used to obtain horizon results, and the subject area of the paper. The majority of the papers treat dynamic problems in inventory management, production planning, capacity expansion, machine replacement, and warehousing. We discuss the relationship of the horizon results with the theory and practice of rolling-horizon procedures and future research directions.

(Multiperiod Problems; Forecast Horizons; Rolling Horizons)

1. Introduction

Studies of forecast, solution, and rolling horizons in dynamic optimization problems are motivated by many important real-world planning problems such as production scheduling and capacity-expansion problems. Dynamic problems are often referred to as multiperiod problems, especially when time is discrete. For multiperiod problems, the decisions for the first period or the first few periods, are usually seen to be of immediate importance to the decision maker. In general, the optimal decisions for these initial periods depend upon the forecast data for periods further into the future. However, because forecasts further into the future are required, the forecasts become less reliable and the cost of forecasting becomes more expensive. Also, there is the increased computational burden associated with solving problems with long horizons. Fortunately, for many dynamic optimization problems in operations management and related

disciplines, the distant forecasts seem to have a diminishing effect on the initial decisions. This diminishing effect allows decision makers, when making their initial decisions, to rely only upon the forecast of the future data for some finite number of periods, which is much less than the future horizon faced by the decision maker.

Horizon research attempts to quantify the diminishing effect, if any, of the future data on the initial decisions. The ideas of forecast/solution/rolling horizons date back to the works of Modigliani and Hohn (1955), Charnes et al. (1955), Johnson (1957), and Wagner and Whitin (1958). Since then, much related research, theoretical as well as applied, has been conducted. Given the importance of horizon issues and the vast amount of associated research, we believe that there is a need for a ready source of reference. To this end, we present a classified bibliography of over 200 published papers (articles in journals and

conference proceedings) and working papers in the horizon literature. Our focus is on papers in operations management, although we do include a sampling of papers from other areas such as economics, finance, marketing, and control engineering. We list the papers in alphabetical order, and categorize each of them on five dimensions [I], [II], [III], [IV], [V], where

- [I] identifies the *horizon type*;
- [II] identifies the *model type*;
- [III] identifies the *sources of horizon*;
- [IV] identifies the *method*;
- [V] identifies the *subject*.

In what follows, we provide a brief description of these five dimensions. Whenever needed, we assume the time to be discrete. An analogous description holds in the continuous-time framework. The purpose here is not to give a detailed discussion of these dimensions, because that would be tantamount to a survey of the field, rather it is to provide some motivation along with a sufficient description of these dimensions for the reader to get some sense of what each of the papers is about. We also direct the reader to some additional references for further details on these dimensions.

I. Horizon Type

As mentioned above, the horizon research focuses on quantifying the diminishing effect of future data on the initial decisions. To formalize the different horizon concepts, let an integer $\Theta \geq 1$ represent the terminal period or the length of the problem horizon. The problem is termed a *finite-horizon* problem if $\Theta < \infty$; otherwise, it is called an *infinite-horizon* problem. Let (t, T) be a pair of real integers, with $1 \leq t \leq T \leq \Theta - 1$. If the optimal decisions in the periods in $[1, t]$ are unaffected by the model parameters (e.g., demands, costs) in the periods in $[T + 1, \Theta]$, then t is called a *decision horizon* (Bes and Sethi 1988) and T is the corresponding *forecast horizon*. In many papers, a decision horizon is also referred to as a *planning horizon*. The literature differentiates between "weak" and "strong" forecast horizons; see Bensoussan et al. (1983) for example. If there are restrictions imposed on the model parameters in $[T + 1, \Theta]$, then T is a

weak forecast horizon. Otherwise, T is a strong forecast horizon. For example, T will be considered a weak forecast horizon if the model requires that there is an upper bound on demands in periods $[T + 1, \Theta]$. A *solution horizon* is the extreme special case of a weak forecast horizon where a complete knowledge of all model parameters in $[T + 1, \Theta]$ is required.

It should be noted that the terms "weak" and "strong" are not sufficiently precise. Their meanings depend on the context. For example, when one obtains a strong forecast horizon in the dynamic lot-size model of Wagner and Whitin (1958), it is so only because no restrictions are imposed beyond the standard assumptions of the model. One of these standard assumptions is that the demands are nonnegative. Clearly, if one allows demands in $[T + 1, \Theta]$ to take negative values, then the forecast horizon obtained under the standard assumptions may no longer be a strong forecast horizon. These considerations led Bes and Sethi (1988) to refine these concepts for general discrete-time dynamic deterministic and stochastic optimization problems. They defined the notion of a forecast of the problem parameters for T periods and a set I of all extensions of that forecast. They were then able to define precisely the concept of a forecast horizon qualified by the set I .

In this bibliography, we do not differentiate between various forecast horizons. We use the term *forecast horizon* (F) to denote any or all of them. Because of its very specialized nature, we do use the term *solution horizon* (S), however, as one of the categories where applicable. Note that a problem that has a finite forecast horizon—strong or weak—also has a finite solution horizon.

A concept related to horizons is that of optimal *myopic* policies discussed in the inventory literature; see, e.g., Heyman and Sobel (1984) and Zipkin (2000). Veinott (1965) is an early reference on this topic. A myopic solution is found by solving a (possibly modified) one-period problem, which is derived easily from the information about the present and future values of the model parameters. In some cases, it can be shown that the solution to the one-period problem is optimal for the multiperiod problem. Because the optimal myopic policy in these problems is calculated

usually under the assumption that the cost and demand parameters are known, these problems can be considered to have weak forecast horizons. The horizons in such problems generally arise because of the nature of costs and constraints on demand. There are several papers that develop optimal and/or near-optimal myopic policies for multiperiod stochastic inventory problems. In this bibliography we limit ourselves to papers that provide optimal myopic policies.

For reasons of efficiency and practicality, rolling-horizon decision making is a common business practice in a dynamic environment (see Naphade et al. 2001 for an application in the steel industry). Under a rolling-horizon procedure, at the beginning of the first period, an R_1 -period problem is solved based on the current state (e.g., inventory/backlog in production-planning problems) and the forecasted data for an appropriately chosen R_1 . Only the current period's decisions become firm. One period later, at the beginning of the second period, the problem state is observed and the forecasts for future periods are updated. A new R_2 -period problem, where R_2 may or may not be chosen to equal R_1 , is solved at the beginning of period two, and the second period's decisions become firm. This procedure is repeated every period, hence the term *rolling horizon*. The number of periods included in the finite-horizon problem is called a *study horizon* (Bean, Smith, and Yano 1987). In most cases, the length of the study horizon is fixed, i.e., $R_1 = R_2$, and so on. The essential aspect is that the horizon gets "rolled over" each period.

Any rolling-horizon procedure may lead to sub-optimal decisions if the study horizon chosen in the rolling-horizon procedure is smaller than the forecast horizon and/or the forecast of the periods within the forecast horizon needs to be updated as time progresses. Many rolling-horizon studies focus on deriving an error bound that provides the maximum possible increase in cost over an infinite-horizon optimal plan when the study horizon used in the rolling-horizon procedure is smaller than the minimal forecast horizon and the forecasts remain unchanged. There are some papers that allow errors in forecasting and include the cost of updating the forecasts (Sethi and Sorger 1991). Moreover, in the Sethi-Sorger model, the

lengths of the successive study horizons are also decisions to be made (on the basis of cost minimization). In this survey we cover rolling-horizon papers that provide some analytical results. We classify them under the term *rolling horizon* (R).

Receding-horizon, moving-horizon, or model-predictive control are terms used in the control-engineering literature for a feedback control technique where the control action is determined by solving an online optimization problem at each step. In other words, a rolling-horizon strategy is used. While there are many papers in control engineering, we have included a few under the classification of rolling horizon (R) for their horizon type.

Finally, we use a category *other* (O) to include studies in which the horizon is defined in some other units than time periods. For example, horizon could be in terms of the number of jobs in scheduling. This category also includes horizons that, while related, are not exactly one of forecast, solution, or rolling horizons.

II. Model Type

A deterministic optimization problem is (of model type) *deterministic* (D) when all of its data (e.g., demand and costs in production-planning problems) in the periods included in the model are deterministic and known. A problem is (of model type) *stochastic* (S) when some of this data are stochastic. Stochastic problems include, of course, those with imperfect information treated, for example, in Hernandez-Lerma and Lasserre (1990).

III. Sources of Horizon

Not all problems have finite solution and/or forecast horizons. The third dimension, sources of horizon, identifies properties of the model that lead to a finite-solution/forecast horizon. For example, Modigliani and Hohn (1955) identified the nonnegativity constraint on inventory and the presence of positive holding costs as two sources of horizons in the convex production-planning problem that they considered. Bean and Smith (1984, 1993) and Sethi and Thompson (2000) provide some discussion of the conditions that lead to horizons. Unfortunately, most papers do not explicitly identify the sources of horizons.

Moreover, it may be quite complicated to identify the complex interactions of the various model parameters that give rise to forecast horizons. Because of this complexity, our classification of this dimension may not be complete. Undoubtedly, the issue of the sources of horizons presents both the challenge and the opportunity for further research.

We next describe the sources of horizons we employ. *Cost structure (CS)*, such as the magnitude of the fixed cost in relation to the inventory holding cost in the dynamic lot-size model of Wagner and Whitin (1958), plays an important role in giving rise to forecast horizons.

The presence of nonnegative marginal costs of production and storage, together with the *constraint (C)* that inventories remain nonnegative, give rise to forecast horizons in Modigliani and Hohn (1955) and Johnson (1957). Lieber (1973) allows backlogging, thereby relaxing the nonnegative inventory constraint in Modigliani and Hohn (1955), but requires the holding cost to have a positive jump in its derivative at the zero inventory level. Constraints such as imposing a lower bound of zero on inventories and an upper bound equal to the warehouse capacity give rise to forecast horizons in the wheat-trading model as discussed in Sethi and Thompson (2000) and Feichtinger and Hartl (1986).

Finite number of decision policies and discounting (FD) can lead to an optimal solution to a T -period problem, which is sufficiently superior to the next-best solution, so that the difference between the two solutions is larger than the small discounted value of the objective function over periods $[T+1, \Theta]$. This results in a forecast horizon of T periods; see Bean and Smith (1984) and Bes and Sethi (1988).

There is a considerable amount of literature in economics dealing with the turnpike property dating back to Dorfman et al. (1958), Morishima (1961), and Radner (1961). The turnpike property, which is related to the asymptotic stability of optimal trajectories, can also be a source of horizons. Accordingly, when the problem horizon Θ is sufficiently large, and initial and terminal states are given, the optimal trajectory will first go quickly from the initial state to an "optimal steady state," stay in that state as long as pos-

sible, and then leave that state some time before the terminal time Θ to reach the specified terminal state. It is this tendency that gives rise to a solution horizon. As an analogy, consider a trip from a location in Dallas to a destination in Indiana. While there are many possible routes, generally, the fastest route is to get to a "turnpike" as quickly as possible, travel on it until reaching a suitable exit to the destination, and then take that exit and drive on to the destination. Whether or not the turnpike property holds depends on the cost structure and other aspects of the model. While this dependence is quite complex, it is fair to say that the existence of the turnpike property requires concavity of the objective function to be maximized and the concavity of the functions describing the dynamics. More precisely, the Hamiltonian function is required to be concave in the state variables and convex in the shadow prices. In this case, the optimal trajectory, which must satisfy the Euler-Lagrange equation of the calculus of variations, converges to an asymptotic optimal steady state given by the saddle point of the Hamiltonian. The Euler-Lagrange necessary condition is related to Pontryagin's Maximum Principle in the optimal-control literature and to the intertemporal efficiency condition in the economics literature. In economic growth models, the optimal trajectory converges to a turnpike represented by the Von Neumann path of the fastest proportional growth. On this path, one can show that the intertemporal efficiency condition holds. That is, the marginal rate of substitution between any two goods regarded as outputs of the previous period must be equal to their marginal rate of substitution as inputs for the next period. In a model of the optimal fish harvest, the turnpike is the steady state path of the maximum sustainable yield.

In this paper, whenever a solution horizon comes about because of the turnpike property, we use the term *turnpike (T)* as the source of that solution horizon. Although we cover operations management papers exhibiting solution horizons due to the turnpike property, we have also chosen to include a few papers from the economics literature, and have classified them with turnpike (T) as their source of horizon. The reader is directed to Carlson and Haurie (1987),

Takayama (1974), and McKenzie (1976) for additional references.

Forecast cost (FC) becomes a source of rolling-horizon decision making if the cost of forecasting becomes more and more expensive as the forecast of the more and more distant future is required; see, e.g., Sethi and Sorger (1991). Morton (1981) has also argued that computation costs and inaccuracies of forecasts, as one proceeds further into the future, make it worthwhile to use a study horizon that is considerably smaller than an exact forecast horizon.

In Markov decision processes, the rate at which finite-horizon optimal policies converge depends on the ergodic property of the underlying Markov chains as well as the discount factor. This category is labeled *ergodicity and/or discounting (ED)*. Since a geometrically distributed random problem-horizon Θ has been shown to be equivalent to a time-varying discount rate (see Ross (1984) and Presman and Sethi (1997)), we will categorize papers (e.g., Iida and Mori 1996) having a random horizon under the horizon source *ED*.

Finally, we provide a category *other (O)* for those studies with some source of horizon different from the sources identified in the above. Survey papers are also included in this category.

IV. Method

The fourth dimension, method, refers to the methodology used in the paper to discover the horizon or to prove certain horizon results. *Dynamic programming (DP)* and *optimal control (OC)* are, by far, the most frequently used methodologies. Less frequently used are *linear programming (LP)*, *nonlinear programming (NLP)*, *branch and bound (BB)*, and *heuristic procedures (H)*. We also use the category *NLP* for papers that use the method of Lagrange multipliers as well as the first-order conditions for finding maxima and minima. We do not include in our classification the complexity status of the algorithms as considered in such papers as Federgruen and Tzur (1991, 1993). The category *other (O)* is for papers whose methods are different from the ones mentioned above, and for survey-type papers.

V. Subject

Finally, the subject identifies the following eight areas of practical applications where horizon results are developed: *inventory management (IM)*, *production planning (PP)*, *scheduling/sequencing (SC)*, *plant location (PL)*, *machine replacement (MR)*, *cash management (CM)*, *capacity expansion (CE)*, and *wheat trading/warehousing (W)*. The wheat-trading problem is a commodity-management problem subject to a warehousing constraint. It is also referred to as the warehousing problem. The inventory-management category includes models that deal with planning lot sizes for individual products or a group of products. Models with both periodic and continuous-review policies are included in this category. The production-planning category includes models that deal with planning the overall production for a plant, given such control variables as overtime, seasonal inventories, backlogging, subcontracting, and hiring and firing of labor. Papers that contain pure mathematical development are classified under *mathematical theory (MT)*. The category *survey (S)* is for survey-type papers. The category *other (O)* is used to include papers with applications that are not specified above.

We now recapitulate the categories under each of the five dimensions:

Section 2 of the paper provides a classified bibliography. Section 3 discusses relationships of the horizon results with current practice and theory dealing with rolling-horizon procedures for solving multiperiod planning problems. Finally, §4 identifies several topics for further research.

2. The Classified Bibliography

The following classified bibliography is presented in tabular form. The first column identifies a paper with the last names of the authors; the second column contains the publication year of the paper. The "Author" column indexes the papers in alphabetical order. The "Year" column arranges the papers by the same authors in chronological order. The full citation of the paper can be found in the References. Columns 3, 4, 5, 6, and 7 correspond to the five dimensions: Horizon Type, Model Type, Sources of Horizon, Method, and Subject, respectively.

Table 1 Categories and Notation

I		Horizon Type
<i>F</i>		Forecast horizons
<i>S</i>		Solution horizons
<i>R</i>		Rolling horizon, error bounds
<i>O</i>		Other
II		Model Type
<i>D</i>		Deterministic
<i>S</i>		Stochastic
III		Sources of Horizon
<i>CS</i>		Cost structure
<i>C</i>		Constraints
<i>FD</i>		Finite number of decisions and discounting
<i>T</i>		Turnpike
<i>FC</i>		Forecast cost
<i>ED</i>		Ergodicity and/or discounting
<i>O</i>		Other
IV		Method
<i>DP</i>		Dynamic programming
<i>LP</i>		Linear programming
<i>OC</i>		Optimal control
<i>BB</i>		Branch and bound
<i>NLP</i>		Nonlinear programming
<i>H</i>		Heuristic
<i>O</i>		Other
V		Subject
<i>IM</i>		Inventory management
<i>PP</i>		Production planning
<i>SC</i>		Scheduling/sequencing
<i>PL</i>		Plant location
<i>MR</i>		Machine replacement
<i>CM</i>		Cash management
<i>CE</i>		Capacity expansion
<i>W</i>		Wheat trading/warehousing
<i>MT</i>		Mathematical theory
<i>S</i>		Survey
<i>O</i>		Other

3. Relationship with Current Practice and Theory

The extensive literature on forecast horizons is intimately related with the common business practice of

solving multiperiod planning problems using a rolling-horizon procedure. Forecast-horizon theory suggests that, in most situations, the data beyond a certain finite horizon have no effect on the optimal decisions in the current period. This idea provides the basis for the rolling-horizon practice. Thus, the rolling-horizon practice can be viewed as a heuristic for implementing the forecast-horizon theory.

Implementing a rolling-horizon procedure requires the selection of a study horizon, appropriate conditions at the end of the study horizon, and a procedure to solve the resulting problem. Managers tend to select the study horizon arbitrarily, and also use some arbitrary values for the ending conditions (for example, zero inventory at the end of the study horizon in a production-planning problem). The arbitrary selection of the study horizon and the ending conditions can leave the firm in an undesirable state in the future. The theoretical "stopping rules" to discover forecast horizons are usually problem specific, and are not easy to develop. To overcome this difficulty, Morton (1981) discusses a number of different possibilities, including:

- (1) Stop when the initial decision has essentially stopped fluctuating;
- (2) Stop when the initial decision is such that the cost of any longer horizon problem, given the initial decision, is within a certain percentage of the minimum cost for the longer horizon problem;
- (3) Stop when the initial decision can be guaranteed to be within a certain percentage of the optimal decision for any longer horizon problem.

Morton (1981) refers to Case 1 as *apparent* forecast horizon, Case 2 as *near-cost* horizon, and Case 3 as *near-policy* horizon. He comments, "while exact horizons are very desirable, they cannot always be found, and to insist on them may be a luxury we cannot afford before being willing to proceed." Bean, Noon, and Salton (1987) used the idea of "apparent horizons" in solving a cash-management problem. They showed that the proper selection of a study horizon, and using an optimal algorithm to solve the problem, saved the company \$40 million compared to traditional techniques that the managers used for solving the problem. Thus, there is value in using study ho-

Table 2 The Classified Bibliography

Author	Year	Horizon Type	Model Type	Sources of Horizon	Method	Subject
(1) Alden and Smith	1992	R	S, D	FD, ED	O	MT
(2) Andreatta and Mason	1994	F	D	CS	DP	PL
(3) Anupindi et al.	1996	F, S, R	S	CS	O	IM
(4) Aronson	1989	F, S, R, O	D	O	O	S
(5) Aronson and Chen	1986	F, S	D	CS, C	LP	MT
(6) Aronson and Chen	1989	F, S	D	CS, C	LP	MT
(7) Aronson and Chen	1993	F, S	D	CS, C	LP	PP, MT
(8) Aronson et al.	1984	F, S	D	CS, C	LP, O	PP, MT
(9) Aronson et al.	1985	F, S	D	CS, C	LP	MT
(10) Aronson and Thompson	1984	O	D, S	O	O	S
(11) Aronson and Thompson	1985	F, S	D	CS, C	LP	PP, O
(12) Baker	1977	R	D	CS	O	IM
(13) Baker and Peterson	1979	R	D	CS	O	IM
(14) Bastian	1992	F	D	FD	DP, O	IM
(15) Bastian and Volkmer	1992	F	D	CS	DP	PL
(16) Bean et al.	1991	O	D	CS	H	SC
(17) Bean, Birge, and Smith	1987	F, S	D	FD	DP	MT
(18) Bean et al.	1992	F, S	S	ED	BB	MT
(19) Bean et al.	1994	R	D	FD	DP	MR
(20) Bean, Noon, and Salton	1987	R	D	FD	BB	CM
(21) Bean and Smith	1984	F, S	D	FD	O	MT
(22) Bean and Smith	1985	F, S, R	D	FD	DP	CE
(23) Bean and Smith	1993	S	D, S	CS, FD	DP	S
(24) Bean et al.	1990	F	S	O	DP	MT
(25) Bean et al.	1985	F, S	D	FD	DP	MR
(26) Bean, Smith, and Yano	1987	F, S, R	D	FD	DP	IM
(27) Beckmann	1961	S	S	FD	OC	PP
(28) Bensoussan and Proth	1985	S	D	CS	M	IM
(29) Bensoussan et al.	1991	F, S	D	CS	DP	IM
(30) Bernardo	1978	F, S	D	CS, C	NLP	PP
(31) Bes and Sethi	1988	F	S	FD	DP	MT
(32) Bhaskaran and Sethi	1981	F	D	C	OC	W
(33) Bhaskaran and Sethi	1983	F	D	S	O	MT
(34) Bhaskaran and Sethi	1987	F	S	S	O	S
(35) Bhaskaran and Sethi	1988	F	S	CS	DP	IM
(36) Blackburn and Kunreuther	1974	F	D	CS	DP	IM
(37) Blackburn and Millen	1980	R	D	CS	H	IM
(38) Blikle and Łos	1967	F	D	CS	DP	IM, PP, MT
(39) Blocher and Chand	1996	F	D	CS, C	BB	SC
(40) Bylka	1974	F, S	D	CS, FD	DP	IM
(41) Bylka	1978	F, S	D	CS	DP	IM
(42) Bylka	1979	F, O	S	CS, FD	DP, O	PP
(43) Bylka	1982	F, O	D	CS, FD	DP, H	IM, MT
(44) Bylka	1997	S, R, T	D	CS, C	DP	IM
(45) Bylka	1999	F, S, T	D	CS	DP	PP
(46) Bylka and Sethi	1992	F, S	D	CS	DP	IM

Table 2 (cont'd.)

Author	Year	Horizon Type	Model Type	Sources of Horizon	Method	Subject
(47) Bylka et al.	1992	F	D	CS	DP	MR
(48) Carlson et al.	1982	R	D	CS	O	IM
(49) Chand	1982	R	D	CS	H	IM
(50) Chand	1983	F	D	CS	DP	IM
(51) Chand	1988	F	D	CS	DP	PL
(52) Chand et al.	1993	F	D	CS	DP	MR
(53) Chand and Morton	1982	F, S, R	D	CS	DP	CM
(54) Chand and Morton	1986	F	D	CS	DP	IM
(55) Chand and Sethi	1982	F	D	CS	DP	MR
(56) Chand and Sethi	1983	F	D	CS	DP	IM
(57) Chand and Sethi	1990	F	D	CS	DP	IM
(58) Chand et al.	1990	F	D	CS	DP	IM
(59) Chand et al.	1992	F	D	CS, FD	DP	IM
(60) Chand et al.	1996	S, O	D	C	BB	SC
(61) Charnes et al.	1966	F	D, S	CS, ED	LP, O	W
(62) Charnes et al.	1955	D	F	CS	NLP	PP
(63) Cheevaprawatdomrong and Smith	2001	F	S	C	DP, NLP	IM, PP
(64) Chen et al.	1995	F, R	D	CS	DP	IM
(65) Chen and Lee	1995	F, R	D	CS	DP	IM
(66) Daskin et al.	1992	F	S	CS	DP	PL
(67) Denardo and Lee	1991	F, R	D	CS	DP	IM
(68) Dutta	1993	S	D	CS	O	MT
(69) Eppen et al.	1969	F	D	CS	DP	IM
(70) Evans et al.	1989	F	D	CS	DP	IM
(71) Federgruen and Tzur	1993	F, S, R	D	CS	DP, H	IM
(72) Federgruen and Tzur	1994	F, S, R	D	CS	DP, H	IM
(73) Federgruen and Tzur	1995	F, S	D	CS	DP	IM, MT
(74) Federgruen and Tzur	1996	F	D	CS	DP	MT
(75) Garcia and Smith	2000a	F, S, R	D	CS, C	DP, O	PP, MT, O
(76) Garcia and Smith	2000b	F, S	D	CS, C	DP	PP
(77) Goldstein and Mehrez	1996	F	S	FD	H	MR
(78) Grey	1984	S	S	ED	DP	MT, O
(79) Hartl	1986	F	D	CS, C	OC	W
(80) Hartl	1987	F	D	C	OC	W
(81) Hartl	1988	F	D	CS, C	OC	W
(82) Hartl	1989	F	D	C	OC	W
(83) Hartl	1995	F, R	D	C	OC	PP
(84) Haurie and Hung	1976	S	D	T	OC	O
(85) Haurie and Sethi	1984	S	D	O	OC	MT
(86) Heady and Zhu	1994	F	D	CS	DP	IM
(87) Herbon et al.	2000a	O	S	O	OC	MT
(88) Herbon et al.	2000b	O	S	O	OC	MT
(89) Hernandez-Lerma and Lasserre	1988	F	S	FD	DP	MT
(90) Hernandez-Lerma and Lasserre	1990	R	S	O	DP	MT
(91) Hopp	1987	F	S	FD	DP	O
(92) Hopp	1988	R	S	ED	DP	MT
(93) Hopp	1989	F	S	ED	DP	MT
(94) Hopp et al.	1987	F	S	ED	DP	MT

Table 2 (cont'd.)

Author	Year	Horizon Type	Model Type	Sources of Horizon	Method	Subject
(95) Hopp and Nair	1991	F	S	CS	DP	MR
(96) Ignall and Veinott	1969	F	S	CS, C	O	IM
(97) Iida and Mori	1996	F, S, R	S	FD	DP, O	MT
(98) Johnson	1957	F	D	CS	O	PP
(99) Johnson and McClain	1978	F	D	CS	O	PP
(100) Johnson and Thompson	1975	F	S	CS, C	O	IM
(101) Keerthi and Gilbert	1986	R	D	O	OC	MT
(102) Kleindorfer and Kunreuther	1978	F, S	S	CS, FC	OP	PP
(103) Kleindorfer and Lieber	1979	F	D	CS, C	DC	PP
(104) Krawczyk	1979	F, O	D	O	OC	MT
(105) Krawczyk	1992	F, R	S	T, O	DP, OC	MT, O
(106) Krawczyk and Karacaoglu	1993	F, R	S	T, O	DP, OC	CM, O
(107) Kunreuther and Morton	1973	F, S	D	CS, C	O	PP
(108) Kunreuther and Morton	1974	F, S	D	CS, C	O	PP
(109) Lasserre	1986a	F	S	FD	DP, OC	MT
(110) Lasserre	1986b	F	D	FD	DP, OC	MT
(111) Lasserre	1988	S	S	O	OC	MT
(112) Lasserre and Bes	1984	F	S	FD	DP, OC	MT
(113) Lasserre et al.	1986	F	D	CS, C, FD	DP	IM
(114) Lasserre and Roubellat	1984	F	D	CS, C, FD	DP	IM
(115) Lee and Denardo	1986	F, R	D	CS	DP	IM
(116) Lee and Kouvaritakis	2000	R	S	O	LP	MT
(117) Lee and Orr	1977	F, S	D	CS, C	NLP	PP
(118) Lieber	1973	F	D	CS	OC	PP
(119) Loś	1967	F, O	D	CS	DP	MT, S
(120) Loś	1971	F, O	D	T, C, O	DP, LP	O
(121) Lundin and Morton	1975	F, S, R	D	CS	DP	IM
(122) Mayne and Michalska	1990	R	D	O	OC	MT
(123) McClain and Thomas	1977	R	D	CS	LP	PP
(124) McKenzie	1976	S	D	T	DP	MT
(125) Michalska and Mayne	1993	R	S	T, O	DP, OC	MT
(126) Michalska and Mayne	1995	R	S	T, O	DP, OC	MT
(127) Miller	1979	F	D	CS	LP	PP
(128) Modigliani and Hohn	1955	F	D	C, CS	NLP	PP
(129) Morton	1978a	F, S, R	S	CS	O	IM
(130) Morton	1978b	F, S, R	D	CS	DP	IM
(131) Morton	1978c	F, R	D, S	CS	O	PP
(132) Morton	1979	F, S, R	D, S	CS	DP	IM, MT
(133) Morton	1981	F, S, R	D, S	C, FC	O	S
(134) Morton and Dharan	1978	F	D	CS	O	SC
(135) Morton and Pentico	1995	F, S, R	S	CS	O	IM
(136) Morton and Udayabhanu	1988	F, S, R	D	CS	DP	CE
(137) Morton and Wecker	1977	F	S	ED	DP	MT
(138) Nagasawa and Nishiyama	1980	F	D	CS	O	PP
(139) Nagasawa et al.	1982a	F	D	CS	O	PP
(140) Nagasawa et al.	1982b	F	D	CS	O	PP
(141) Nagasawa et al.	1982c	F	D	C	NLP	PP

Table 2 (cont'd.)

Author	Year	Horizon Type	Model Type	Sources of Horizon	Method	Subject
(142) Nagasawa et al.	1983	R	D	CS	O	PP
(143) Nagasawa et al.	1985a	F	D	CS	O	PP
(144) Nagasawa et al.	1985b	R	D	CS	O	PP
(145) Nair	1995	F	S	FD	DP	MR
(146) Nair and Hopp	1992	F	S	CS, FD	DP	MR
(147) Naphade et al.	2001	R	D	C	H	SC
(148) Nerlove and Arrow	1962	S	D	T	OC	O
(149) Ovacik and Uzsoy	1995	S, O	D	C	H	SC
(150) Park et al.	1993	O	D, S	CS	O	MT
(151) Pekelman	1974	F	D	C	OC	PP
(152) Pekelman	1975	F	D	CS, C	OC	PP
(153) Pekelman	1979	F	D	C	OC	PP
(154) Primbs and Nevistic	2000	R	D	O	OC	MT
(155) Proth	1984	F, S	D	CS	DP, O	IM
(156) Rajagopalan	1992	F, R	D	CS	LP	CE
(157) Rajagopalan	1994	F	D	CS	LP	CE
(158) Ravn	1987	F	D	CS, C	OC	MT, O
(159) Rempala	1979	F	D	C	OC	PP, MT
(160) Rempala	1984	F, S	D	C	OC	PP, MT
(161) Rempala	1985	F	D	C	OC	PP, MT
(162) Rempala	1986	F, S	S	C	DP	IM, MT
(163) Rempala	1989a	F	D	C	OC	W
(164) Rempala	1989b	F	D	CS, C	OC	PP
(165) Rempala	1989c	F	S	CS, C	DP	SC, MT
(166) Rempala	1990	F	D	CS, C	OC	PP
(167) Rempala	1991	F	D	C	OC	MT
(168) Rempala	1999	F	D	CS, C	OC	PP, W
(169) Rempala and Sethi	1988	F	S	C	OC	IM
(170) Rempala and Sethi	1992	F	D	C	OC	MT
(171) Richter and Weber	1994	F	D	CS	O	IM
(172) Ryan	1998	R	D	CS	O	CE
(173) Ryan and Bean	1989	F	D	FD	DP	MT
(174) Ryan et al.	1992	F, S	D	O	O	MT
(175) Sandbothe and Thompson	1990	F	D	CS	DP	IM
(176) Sandbothe and Thompson	1993	F	D	CS	DP	IM
(177) Saydam and McKnew	1987	F	D	CS	DP	IM
(178) Schochetman and Smith	1989	F, S	D	CS	DP, NLP	PP
(179) Schochetman and Smith	1991	F, S	D	FD	O	MT
(180) Schochetman and Smith	1992	F, S	D	CS, C	LP, NLP	PP
(181) Schochetman and Smith	1998	F, S	D	CS, C	DP	MT
(182) Schwarz	1977	F, R	D	CS	H	DL
(183) Sethi	1971	F	D	CS	O	CM
(184) Sethi	1973	S	D	T	OC	O
(185) Sethi	1974	S	D	T	OC	O
(186) Sethi	1977	S	D	T	OC	MT
(187) Sethi	1978	S	D	T	OC	O
(188) Sethi	1984	F	D	O	O	S
(189) Sethi	1987	F	D, S	S	O	S

Table 2 (cont'd.)

Author	Year	Horizon Type	Model Type	Sources of Horizon	Method	Subject
(190) Sethi	1990	F	D, S	S	O	S
(191) Sethi and Bes	1988	F	S	FD	DP	MT
(192) Sethi and Bhaskaran	1985	F	S	FD	DP	MT
(193) Sethi and Chand	1979	F	D	CS	DP	MR
(194) Sethi and Chand	1981	F	D	CS	DP	IM
(195) Sethi and Sorger	1989	F	S	FD	OC	MT
(196) Sethi and Sorger	1991	R	S	CS	DP	MT
(197) Sethi and Thompson	1982	F	D	C	OC	W
(198) Shapiro	1968	F	S	ED	DP	MT
(199) Shapiro and Wagner	1967	F	D	CS	DP	IM, CE, O
(200) Smith	1981	S	D	CS, T	DP	CE
(201) Smith and Zhang	1998	F, S	D	CS	NLP	PP
(202) Stadler	2000	F, R	D	CS	DP	IM
(203) Sung and Lee	1994	F, R	D	CS	DP	IM
(204) Teng et al.	1984	F	D	C	OC	PP
(205) Tesfatsion	1981	R	S	O	OC	O
(206) Thomas	1970	S	D	CS	DP	IM
(207) Thompson and Sethi	1980	S	D	T	OC	PP
(208) Thompson et al.	1984	F	D	C	OC	PP
(209) Tzur	1996	F	D	CS	DP, H	IM
(210) Udayabhanu and Morton	1988	F, S	D	CS, FD	DP	CE
(211) Vanthienen	1973	F	D	CS, C	OC	PP
(212) Veinott	1965	F	S	CS, C	O	IM
(213) Wagner and Whitin	1958	F	D	CS	DP	IM
(214) Wemmerlov and Whybark	1984	F	S	CS	O	IM
(215) White	1996	F	S	C, FD	DP, O	MT
(216) Willke and Miller	1978	S	S	FD	OC	PP
(217) Zabel	1964	F	D	CS	DP	IM
(218) Zangwill	1969	F	D	CS	DP	IM

horizons based on even simple stopping rules in place of completely arbitrary study horizons.

There is an extensive amount of literature on the dynamic lot-sizing problem of Wagner-Whitin (WW) and many variations of it. This includes both the optimal algorithms and the horizon results. Federgruen and Tzur (1994) developed some results to address the "nervousness" problem in MRP applications. They also showed that forecast horizons for these problems tend to be small, that is, include only a small number of orders. Yet, Hopp and Spearman (2001) report that companies actually do not use these results. They mention, "Interestingly, we know of no commercial MRP package that actually uses the WW algorithm. The reasons usually given are that it

is too complicated or that it is too slow" (p. 124). MRP systems use various heuristics to solve the lot-sizing problems using study horizons that are appropriate for the heuristics. These heuristics use the zero-inventory paradigm (zero inventory at the time of receiving an order) in Wagner-Whitin, modified to allow for safety stocks, lead times, and fixed lot sizes. Note that although tedious to solve by hand, the Wagner-Whitin algorithm can be implemented easily on a computer and solved efficiently. Thus, given the easy availability of fast computers, the reasons given for using heuristics in place of optimal algorithms to solve the dynamic lot-sizing problems are not convincing. In the words of Hopp and Spearman (2001), a more likely reason may be found in the observation

that "people would rather live with a problem they cannot solve than accept a solution they do not understand" (p. 124).

4. Topics for Future Research

Although the horizon research has seen an impressive output over the last 50 years, there are important issues yet to be addressed. We feel that one of the most important issues is to develop a better understanding of the sources of horizons in different problems. In some cases, there may be individual factors responsible for the horizon results, while in others, horizons may result from a complex interaction of many different factors. Unfortunately, not many authors explicitly identify the sources of horizons for their problems. Identifying the sources of horizon in a problem is difficult, but it presents an opportunity for further research.

In a rolling-horizon, decision-making environment, a manager should ideally choose the length of the study horizon that minimizes the cost of data collection, the cost of computation, and any increase in the objective function cost due to a suboptimal first decision. The manager may even have to choose a data-collection method if there is a trade-off between the cost of data collection and the data accuracy. The relationship of the data-collection cost, data accuracy, and the computational cost to the length of study horizon is an important research problem. Morton (1981) also identified this to be an important problem. The results of such research could be very valuable to practitioners in their day-to-day decision making. While this issue has been treated in Kleindorfer and Kunreuther (1978) and Sethi and Sorger (1991) to some extent, it deserves much more attention. In particular, Sethi and Sorger incorporate forecast costs in their model to define a concept of the rolling-horizon optimality. While this is the only framework available in the literature, the dynamic-programming methodology developed in Sethi and Sorger is impractical, due to the curse of dimensionality. To develop efficient algorithms that would obtain rolling-horizon, suboptimal solutions with some prespecified deviation from optimality would be of great value. What we mean here is that research should be directed to

develop a framework of rolling-horizon ϵ optimality, along with specific algorithms that are much more efficient than dynamic programming. An algorithm that yields a solution with the desired accuracy takes ϵ as an input; although the lower the value of ϵ , the larger the computational burden. Such efforts would be analogous to the research that has resulted in ϵ -polynomial time-approximation algorithms as well as polynomial-time algorithms with known worst-case bounds for many problems known to be NP-hard.

Practitioners may prefer to use fixed-length study horizons in place of variable-length study horizons. Fixed-length study horizons do not require use of forward algorithms, and may be easier to implement. It would be useful to have an optimization framework to find a fixed-length study horizon. This framework will also need to consider forecasting cost, forecast quality, the cost of data collection, and any increase in the objective-function cost due to possible suboptimal first decisions. The difference is that the increase in the objective-function cost will need to be calculated based on some expectation of all future costs and demands.

Owing to consideration of forecast costs, the chosen study horizon in a rolling-horizon procedure may be shorter than the minimal forecast horizon. Thus, optimality cannot be guaranteed. Lee and Denardo (1986), Denardo and Lee (1991), and Chen et al. (1995) used horizon results to develop error bounds on the initial decision when the study horizon is shorter than the forecast horizon. Furthermore, it would be desirable to develop strategies that would provide decisions with a "good" performance in a rolling schedule. There have been a few proposed strategies in the literature, including those of specifying terminal conditions such as terminal inventory at the end of the study horizon on a rolling basis (Baker and Peterson 1979, Chand and Morton 1986, McClain and Thomas 1977) and of extending demand forecast beyond the study horizon (Carlson et al. 1982, Russel and Urban 1993, Stadtler 2000). However, their findings (most of them are by simulation) seem to confirm observations made by Baker (1977) that the performance of optimal study-horizon decisions in a rolling-schedule environment depend in

very complex ways on factors such as the length of study horizon, cost structure, and demand patterns. Also, the effect of forecast errors in future demands on the performance of these algorithms is not well understood. In most problems of practical interest, the variance of forecast errors is increasing over time. Future research on these important practical issues is very desirable.

Yet another important research problem is to develop a better understanding of the effect of the nature of costs and demand patterns on the length of decision and forecast horizons. This may lead to simple rules to guide practitioners in selecting the appropriate study horizons in rolling-horizon, decision-making environments. For example, Lundin and Morton (1975) and Schwarz (1977) show that 5-EOQ's worth of study horizon gives solutions within 1% of optimality under the assumptions of stationary cost and demand parameters. Chand et al. (1990) and Chand et al. (1992) extend this result to the situation when the cost and demand parameters are assumed to be stationary only over the forecast horizon. Moreover, they show the existence of a finite forecast horizon under certain a priori verifiable conditions. There is need for developing similar existence results for other models in the horizon literature that would, given the model data, determine whether or not a finite forecast horizon exists.

While horizon results have been obtained for many operations management problems, there are many other dynamic operations management problems with the potential of exhibiting horizons, and for which no horizons results have been obtained. Particular examples are the dynamic lot-sizing problem with concave costs and perishable inventory treated by Hsu (2000) and some variants of the capacity-expansion problem treated by Rajagopalan (1992, 1994). These authors have developed polynomial-time algorithms for their problems without looking at their horizon aspects. Similarly, it may be possible to extend the horizon results obtained for some scheduling problems with dynamic job arrivals to problems studied by Keskinocak et al. (2001) and Kapuscinski and Tayur (2000) that coordinate scheduling with lead-time quotation.

5. Concluding Remarks

This paper has provided a classified bibliography of over 200 research papers in the horizon literature. While a majority of papers treat deterministic, single-dimensional, multiperiod planning problems in operations management, the real-life problems are multidimensional (multiproduct, multimachines, etc.) and beset with huge uncertainties. Managers usually face forecasts with increasing randomness over time; for example, monthly forecast could vary by a factor of two a few months out or a factor of five a year out. For such complex problems, it would be very difficult, if not impossible, to obtain forecast horizons, even though they are believed to exist in these problems under reasonable conditions. So the important managerial question is how to make the horizon results accessible as well as useful to managers.

Horizon research justifies the usual practice of rolling-horizon decision making; in a similar way the optimality of (s, S) policies established for single-product, stochastic inventory problems provides a heuristic justification for two-bin policies used in many real-life, multiproduct inventory problems. Research on the sources of horizons and error bounds should yield useful heuristics for decision making in practice. In particular, these results should provide guidelines for selecting appropriate study horizons in a rolling schedule. Already in the discounted cost and/or ergodic cases, it is possible to provide study horizons that are ϵ optimal. Typically, these study horizons might be too long to be of practical use. Suggested future research on the sources of horizons, bounds on initial decisions for a given study horizon, and efficient algorithms to provide near-optimal, rolling-horizon schedules should provide useful insights and heuristics in shortening the lengths of these study horizons in particular, and for practical decision making in complex multiperiod operations management problems in general.

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An earlier version of the paper was circulated to a number of leading authors in the horizon area in order to obtain their comments and feedback. Most of them responded to our request. We appreciate their comments and feedback.

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